

Dynamics and Vibroacoustics of Machines (DVM2014)

**Acoustoelastic behaviour of double-layer plates accounting
internal damping of layer materials**

V.N. Paimushin*

*Kazan National Research Technical University named after A.N.Tupolev – KAI, K.Marx Str., 10, Kazan, 420111, Russian Federation***Abstract**

In this paper, the coupled problem of planar acoustic wave propagation through the double-layer composite plate containing the second layer from damping material with high logarithmic decrement is examined. Aerohydrodynamic interaction between plate and external acoustic environment is defined by three-dimensional wave equations, whilst mechanical behaviour of double-layered plate is examined with model based on classical Kirchhoff-Love's hypothesis. Exact analytical solutions are given for plates with simply supported edges. On the basis of given solutions, parameters for second layer, which lead substantially damping of plate vibrations are found in the case of acoustic loading at resonant modes.

© 2015 Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Peer-review under responsibility of organizing committee of the Dynamics and Vibroacoustics of Machines (DVM2014)

Keywords: Double-layered plate; acoustoelasticity; internal damping; logarithmic decrement; wave equation; acoustic wave; analytical solution; resonance; vibration damping.

1. Introduction

The level of allowable vibrations in any structure is determined by many factors such like its effect to the strength of the structure and its separate elements, to the structural operability, structural health, human safety and excitation of mounted equipments. In the assessment of the strength characteristics of any structure, one of the most dangerous regimes in structural dynamic deformation is resonance, which is realized when the frequencies of the structure's natural oscillations coincide with a frequency of the external harmonic effects. In such loading regime, the amplitude levels of stresses and deformations in structure increase dramatically. Their correct and reliable

* Corresponding author. Tel.: +7-903-306-64-84
E-mail address: vpajmushin@mail.ru

theoretical assessment with the needed accuracy for practical purposes requires proper account of damping properties of structural materials caused by internal friction.

Up to now, there are extensive scientific studies on the methods of damping properties determination and on building appropriate mathematical models for their correct descriptions. In general, traditional structural materials (metals and their alloys, advanced composite materials) possessing very high level of elastic and strength properties gives low damping characteristics. Hence, to maximize damping parameters of thin-walled structures made of these materials, their elements are designed in the form of multilayer structures with consecutive placement of high damping materials between the rigid layers throughout the thickness of structure. Such elements are now widely used in the structures of aircrafts and various surface vessels, automobile constructions, civil and industrial buildings, etc., in which, as a damping layer, various elastomers (rubber) are used.

The level of allowable structural vibrations should be limited also by the noise formed in the acoustics environment, which surrounds the structure, by its dynamical interaction with deforming structure.

Mainly, vibration of mechanical systems is dealt by specialists in the fields of solid mechanics, dynamic and strength of machinery, devices and equipment, aircraft and ship strength not taking into account sufficiently the noise produced by the structure during its dynamical deformations, yet the onset and propagation of noise is dealt by the specialist of acoustics. In the last decades of aircraft industry the problem of noise reduction has led to grow up a new scientific field – aeronautical acoustics associated with the acoustics of aircraft and includes aeroacoustics and structural acoustics [1, 2 et al.]. The last one contains scientific discipline at the junction of acoustics and dynamic of elastic systems, which studies the mechanisms of sound propagation by design of the equipment, sound radiation of these structures and other issues. The literature devoted to the study of these issues is quite a lot. Nevertheless, the obtained results in this area should be considered as highly "focused" meaning that coverage is only a narrow class of the simplest elements of thin-walled structures.

As an addition to the discussion above, in the second half of the last century, a scientific field was born in mechanics related to the study of steady and unsteady interaction of acoustic waves with solid deformable bodies and thin-walled structural elements. Studies in this field continue attracting the attention of researchers with actuality, complexity and diversity of the phenomena inherent in the interactions among different physical nature. Related to this research direction, up to now, aerohydroelasticity problems of thin-walled structures (in particular shells) covered in a number of monographs and literature reviews [3-6 et al.].

However, these papers did not consider the questions of sound waves formation and theoretical study of sound insulation and sound absorption by various deformable bodies. These issues are not investigated by researchers up to now, although all handbooks or manuals, devoted to this issue and covering various multilayer structures, point out their good sound insulation and sound absorption properties [7, 8 etc.]. In practice, these properties have been studied mainly experimentally and the theoretical researches on these properties are mostly based on simplified equations of mechanics of multilayer structures since there is no theoretical basis developed to deal with these problems. As an example, noise in ground vehicles (i.e. automobiles) and aircraft structures is reduced by sticking some special coatings made from rubber-like materials on load bearing elements of construction, which are called by designers as sound isolation layers. However, these coatings in practice do not have any sound isolation properties, but having high damping properties, they reduce significantly the amplitudes of deformation and deflections in structural elements during their loading in resonance regimes, which lead, as a result, the reduction of sound pressure in structure's interior.

Moreover, further will be shown in present study that using special coating materials with high damping ratios leads dramatic reductions in the level of cyclic stresses formed in structural elements and as a consequence dramatic increase in durability of structure.

2. Problem statement and solution procedure

Let's consider a deforming body composed of two thin layers having thicknesses t_1, t_2 and made from orthotropic materials. The considered double layer structure belongs to the orthogonal cartesian coordinate system Ox_1x_2z

whose coordinate plane $z=0$ coincide with the mid-plane σ of the first layer and x_1, x_2 axes are aligned with the orthotropy axes of layer materials.

Let u_1, u_2, w – displacements of points on σ plane in the directions of x_1, x_2, z . According to Kirchhoff-Love hypothesis in moderate bending of plate, displacement and deformation components in any point can be defined by relationships

$$\begin{aligned} U_i &= u_i - zw_{,i}, \quad \varepsilon_{ij} = \varepsilon_{ij}^0 - zw_{,ij}, \\ -t/2 \leq z \leq (t_1/2 + t_2), \\ 2\varepsilon_{ij}^0 &= u_{i,j} + u_{j,i} + w_{,i}w_{,j}, \end{aligned} \quad (1)$$

where partial derivatives with respect to the x_i coordinates are designated by standard notation.

If layer materials have viscoelastic properties, then for the description of dynamic deformation process altering with time τ through the harmonic law with circular frequency ω , stress components of k -th layer can be related with deformation components in (1) with constituent relationships as follow

$$\begin{aligned} \sigma_{11}^{(k)} &= \frac{E_1^{(k)}}{1 - \nu_{12}^{(k)}\nu_{21}^{(k)}} \left(1 + \frac{\delta_1^{(k)}}{\pi\omega} \frac{\partial}{\partial \tau} \right) \left[\varepsilon_{11}^0 + \nu_{21}^{(k)} \varepsilon_{22}^0 - z(w_{,11} + \nu_{21}^{(k)} w_{,22}) \right] \overline{1,2}, \\ \sigma_{12}^{(k)} &= 2G_{12}^{(k)} \left(1 + \frac{\delta_{12}^{(k)}}{\pi\omega} \frac{\partial}{\partial \tau} \right) (\varepsilon_{12}^0 - zw_{,12}), \end{aligned} \quad (2)$$

which are generalization of relationships [9] used in article [10] for isotropic materials.

Hereinafter: $-t_1/2 \leq z \leq t_1/2$ for $k=1$; $t_1/2 \leq z \leq t_1/2 + t_2$ for $k=2$; $\delta_i^{(k)}, \delta_{12}^{(k)}$ – logarithmic decrements (LD) in axial and shear deformations of k -th layer's material, $E_1^{(k)}, G_{12}^{(k)}, \nu_{12}^{(k)}, \nu_{21}^{(k)} = E_2^{(k)}\nu_{12}^{(k)}/E_1^{(k)}$ – elastic moduli of first and second kinds and Poisson's ratios.

In terms of internal forces and moments of mid-plane σ ,

$$\begin{aligned} T_{sj} &= \int_{-t_1/2}^{t_1/2} \sigma_{sj}^{(1)} dz + \int_{t_1/2}^{t_1/2+t_2} \sigma_{sj}^{(2)} dz, \\ M_{sj} &= - \int_{-t_1/2}^{t_1/2} \sigma_{sj}^{(1)} z dz - \int_{t_1/2}^{t_1/2+t_2} \sigma_{sj}^{(2)} z dz, \end{aligned} \quad (3)$$

the equation of motion for infinitessimally small element cut from plate built on approximations can be written as follow

$$T_{s1,1} + T_{s2,2} - q\ddot{u}_1 = 0; \quad q = r_1 t_1 + r_2 t_2; \quad s=1,2; \overline{1,2}, \quad (4)$$

$$M_{11,11} + 2M_{12,12} + M_{22,22} + (T_{11}w_{,1} + T_{12}w_{,2})_{,1} + (T_{12}w_{,1} + T_{22}w_{,2})_{,2} + q\ddot{w} - p = 0, \quad (5)$$

where points above symbols shows partial derivatives with respect to τ , r_k – material density of k -th layer, p – external aerohydrodynamic load which is to be found. Constructed equations in (4), (5) can be expressed through displacement components u_1, u_2 and w using physical relationships;

$$T_{11} = B_1 \varepsilon_{11}^0 + \tilde{B} \varepsilon_{22}^0 - \frac{B_1^{(2)} H}{2} (w_{,11} + \nu_{21}^{(2)} w_{,22}); \quad T_{12} = 2B_{12} \varepsilon_{12}^0 - B_{12}^{(2)} H w_{,12}; \quad \overline{1,2} \quad (6)$$

$$M_{11} = -\frac{B_1^{(2)} H}{2} (\varepsilon_{11}^0 + \nu_{21}^{(2)} \varepsilon_{22}^0) + D_1 w_{,11} + \tilde{D} w_{,22}; \quad M_{12} = -B_{12}^{(2)} H \varepsilon_{12}^0 + 2D_{12} w_{,12}; \quad \overline{1,2}, \quad (7)$$

which lead from the relationships (3) by applying relationships (1), (2) and introducing the notation

$$\begin{aligned} H &= t_1 + t_2; \quad B_1^{(k)} = \frac{E_1^{(k)} t_k}{1 - \nu_{12}^{(k)} \nu_{21}^{(k)}} \left(1 + \frac{\delta_1^{(k)}}{\pi \omega} \frac{\partial}{\partial \tau} \right); \quad B_{12}^{(k)} = G_{12}^{(k)} t_k \left(1 + \frac{\delta_{12}^{(k)}}{\pi \omega} \frac{\partial}{\partial \tau} \right); \\ B_1 &= B_1^{(1)} + B_1^{(2)}; \quad \tilde{B} = B_1^{(1)} \nu_{21}^{(1)} + B_1^{(2)} \nu_{21}^{(2)}; \quad B_{12} = B_{12}^{(1)} + B_{12}^{(2)}; \quad \overline{1,2} \\ D_s^{(1)} &= \frac{B_s^{(1)} t_1^2}{12}; \quad D_s = D_s^{(1)} + D_s^{(2)}; \quad s = 1, 2, \\ D_s^{(2)} &= \frac{B_s^{(2)} t_2^2}{12} \mathfrak{a}; \quad D_{12}^{(1)} = \frac{B_{12}^{(1)} t_1^2}{12}; \quad D_{12}^{(2)} = \frac{D_{12}^{(2)} t_2^2}{12} \mathfrak{a}; \quad \tilde{D} = D_1^{(1)} \nu_{21}^{(1)} + D_1^{(2)} \nu_{21}^{(2)}; \\ D_{12} &= D_{12}^{(1)} + D_{12}^{(2)}; \quad \mathfrak{a} = 4 + \frac{3t_1^2}{t_2^2} + 6 \frac{t_1}{t_2}. \end{aligned} \quad (8)$$

Further we will be restricted by investigations of dynamic deformation process of double-layer structure, which will be handled at two consecutive stages: at first stage, which is static one, a nonhomogeneous stress field with respect to x_1, x_2 coordinates is generated in plate, which is equivalent to N_{sj} ; at the second stage, in the vicinity of stress-strain state of the first stage, cyclic deformation process is realized by the formation of such a plate bending stresses and strains, which have little effect on the generated forces N_{sj} of the first stage.

In the perspective of the equality assumptions ($T_{sj} = N_{sj}$) that is valid for small displacements, using the relationships (6), we obtain the strain-displacement relationships:

$$\begin{aligned} \varepsilon_{11}^0 &= \frac{H}{2B_d} (G_1 w_{,11} - \tilde{G}_1 w_{,22}) + \frac{1}{B_d} (B_2 N_{11} - \tilde{B} N_{22}); \quad \overline{1,2}, \\ 2\varepsilon_{12}^0 &= \frac{B_{12}^{(2)} H}{B_{12}} w_{,12} + \frac{N_{12}}{B_{12}}, \end{aligned} \quad (9)$$

where

$$G_1 = B_1^{(2)} B_2 - \nu_{12}^{(2)} B_2^{(2)} \tilde{B}; \quad \tilde{G}_1 = B_2^{(2)} \tilde{B} - \nu_{21}^{(2)} B_1^{(2)} B_2; \quad B_d = B_1 B_2 - \tilde{B}^2. \quad (10)$$

In the view of dependencies (9) relationships in (7) take the form

$$\begin{aligned} M_{11} &= d_1 w_{,11} + \tilde{d}_1 w_{,22} + g_1 N_{11} + \tilde{g}_1 N_{22}; \quad \overline{1,2}, \\ M_{12} &= 2d_{12} - g_{12} N_{12}, \end{aligned} \quad (11)$$

in which the following designations are introduced:

$$d_1 = D_1 - \frac{B_1^{(2)} H^2}{4B_d} (G_1 - \nu_{21}^{(2)} \tilde{G}_2), \quad \tilde{d}_1 = \tilde{D} + \frac{B_1^{(2)} H^2}{4B_d} (\tilde{G}_1 - \nu_{21}^{(2)} G_2) \overline{1,2};$$

$$g_1 = -\frac{B_1^{(2)} H^2}{2B_d} (B_2 - \nu_{21}^{(2)} \tilde{B}), \quad \tilde{g}_1 = \frac{B_1^{(2)} H}{2B_d} (\tilde{B} - \nu_{21}^{(2)} B_1) \overline{1,2}; \quad d_{12} = D_{12} - \frac{(B_{12}^{(2)} H)^2}{4B_{12}}, \quad g_{12} = \frac{B_{12}^{(2)} H}{2B_{12}}. \quad (12)$$

Substituting the relationships (11) into the equation (5), we get governing equation of motion, defined by the function w , which can be written in the case $N_{sj} = T_{sj} = const$ as follow

$$d_1 w_{,1111} + (\tilde{d}_1 + \tilde{d}_2 + 4d_{12}) w_{,1122} + d_2 w_{,2222} + N_{11} w_{,11} + 2N_{12} w_{,12} + N_{22} w_{,22} - p - q \ddot{w} = 0. \quad (13)$$

Boundary and initial conditions for the constructed governing equations (13) are written in the same way as in classical theory of plates based on Kirchhoff-Love hypothesis.

Assume that the plate has a rectangular shape in plan view, with dimensions a, b in the directions of the axes $x = x_1, y = x_2$, and surrounded in two sides by acoustic environments "1" and "2", occupying half planar spaces V_1 and V_2 , bounded by $z = 0$ plane. Action of planar harmonic incident waves to the plate defined by pressure p_* and frequency ω . As a result of its interaction with plate in the surrounding half-spaces V_1 and V_2 acoustic waves are excited in the form of reflected and radiated waves in first half-space and radiated in second one. These waves are defined, relative to the velocity potentials Φ_*, Φ_1, Φ_2 , by wave equations, which are written in approximated way as follow (hereinafter $k=1,2$)

$$\Phi_{*,zz} - c_1^{-2} \ddot{\Phi}_* = 0, \quad (14)$$

$$\Phi_{k,xx} + \Phi_{k,yy} + \Phi_{k,zz} - c_k^{-2} \ddot{\Phi}_k = 0,$$

where c_k – sound velocity in mediums «1» and «2». Through the function Φ_*, Φ_k pressures p_*, p_k and velocity components v_z^*, v_z^k can be determined in halfspaces V_1, V_2 by the relationships (ρ_k – density of mediums «1» and «2»)

$$p_* = -\rho_1 \dot{\Phi}_*, \quad v_z^* = \Phi_{*,z}, \quad (15)$$

$$p_k = -\rho_k \dot{\Phi}_k, \quad v_z^k = \Phi_{k,z}.$$

In this case, aerohydrodynamic load p acting on plate will be

$$p = (p_* + p_1 - p_2) \Big|_{z=0}, \quad (16)$$

and velocity components v_z^*, v_z^k obtained from unseparated interaction of acoustic medium with plate must satisfy the conditions

$$\dot{w} = (v_z^1 + v_z^2) \Big|_{z=0}, \quad \dot{w} = v_z^2 \Big|_{z=0}. \quad (17)$$

Following representations are valid for the harmonic waves (i – imaginary unit),

$$\Phi_* = \tilde{\Phi}_* e^{i\omega t}, \quad \Phi_k = \tilde{\Phi}_k e^{i\omega t}, \quad w = \tilde{w} e^{i\omega t}, \quad (18)$$

and for the plate with simply supported edge conditions at $x=0, x=a, y=0, y=b$, solution of equation (13) will have the following form

$$w = \sum_{m,n=1,3,\dots}^{M,N} \tilde{w}_{mn} \sin \lambda_m x \cdot \sin \lambda_n y \cdot e^{i\omega\tau} \quad (19)$$

where $\lambda_m = m\pi/a$, $\lambda_n = n\pi/b$; $m, n = 1, 3, \dots, M, N$.

In the view of equation (17) solution of the last two equations (14) must be sought in the form

$$\Phi_k = \sum_{m,n=1,3,\dots}^{M,N} \tilde{\Phi}_k^{mn} \sin \lambda_m x \cdot \sin \lambda_n y \cdot e^{i\omega\tau} \quad (20)$$

Taking into account the representations (18)–(20) equation (14) rearranged into the form

$$\tilde{\Phi}_{*,z} + k_1^2 \tilde{\Phi}_* = 0, \quad \tilde{\Phi}_{k,zz} - (\kappa_k^{mn})^2 \tilde{\Phi}_k^{mn} = 0 \quad (21)$$

where

$$\kappa_k^{mn} = \sqrt{\lambda_m^2 + \lambda_n^2 - k_k^2}, \quad k_k = \omega^2 / c_k^2. \quad (22)$$

The first equation for incident wave in virtue of (18) has solution of the form $\tilde{\Phi}_* = A_* e^{-i(k_1 z - \omega\tau)}$, by which, regarding to the relationships (15) we get into the dependencies below

$$\begin{aligned} p_*|_{z=0} &= \tilde{p}_*|_{z=0} e^{i\omega\tau}, \quad \tilde{p}_*|_{z=0} = -i\rho_1 \omega A_*, \\ v_z^* &= \tilde{v}_z^*|_{z=0} e^{i\omega\tau}, \quad \tilde{v}_z^*|_{z=0} = -ik_1 A_*, \end{aligned} \quad (23)$$

where A_* – integration constant, which the pressure and the velocity of incident waves are defined through.

Solutions of the rest of the equations in (21) depend on the sign of the value $(\kappa_k^{mn})^2$. For $(\kappa_k^{mn})^2 > 0$ we obtain the solutions in the form

$$\tilde{\Phi}_1^{mn} = B_1^{mn} e^{\kappa_1^{mn} z}, \quad \tilde{\Phi}_2^{mn} = A_2^{mn} e^{-\kappa_2^{mn} z}, \quad (24)$$

and for $(\kappa_k^{mn})^2 < 0$

$$\begin{aligned} \tilde{\Phi}_1^{mn} &= B_1^{mn} e^{i\tilde{\kappa}_1^{mn} z}, \quad \tilde{\Phi}_2^{mn} = A_2^{mn} e^{-i\tilde{\kappa}_2^{mn} z}, \\ (\tilde{\kappa}_k^{mn})^2 &= -(\kappa_k^{mn})^2, \end{aligned} \quad (25)$$

where B_1^{mn}, A_2^{mn} being integration constants.

Representing the constant A_* by Fourier series as follow:

$$A_* = \sum_{m,n=1,3,\dots}^{M,N} A_* f_{mn} \sin \lambda_m x \cdot \sin \lambda_n y, \quad (26)$$

$$f_{mn} = \frac{16}{\pi^2 mn}$$

conditions in (17) is held for the obtained solutions in (24), (25) and relationships in (19), (23). As a result, for the determination of integration constants we get the following equations,

$$B_1^{mn} = \frac{i\omega}{\mathfrak{A}_1^{mn}} \left(\frac{f_{mn}}{c_1} A_* + \tilde{w}_{mn} \right), \quad (27)$$

$$A_2^{mn} = -i \frac{\omega}{\mathfrak{A}_2^{mn}} \tilde{w}_{mn}.$$

Hereinafter, the values of κ_k^{mn} can be found by the formula in (22). If $\lambda_m^2 + \lambda_n^2 - k_k^2 > 0$ is held, then the equation $\kappa_k^m = i\sqrt{k_k^2 - \lambda_m^2 - \lambda_n^2}$ is applied to determine the value while

$$\lambda_m^2 + \lambda_n^2 - k_k^2 < 0. \quad (28)$$

Using the relations in (27) in accordance with the relations in (15), (18), (20), (23) – (26) we can derive the expression for p as follow

$$p = \sum_{m,n=1,3,\dots}^{M,N} \tilde{p}_{mn} \sin \lambda_m x \cdot \sin \lambda_n y \cdot e^{i\omega\tau}, \quad (29)$$

where

$$\tilde{p}_{mn} = -i\rho_1\omega f_{mn} \left(1 + i \frac{\omega}{c_1\kappa_1^{mn}} \right) A_* + \varphi_{mn} w_{mn}, \quad (30)$$

$$\varphi_{mn} = \frac{\rho_1\omega^2}{\kappa_1^{mn}} + \frac{\rho_2\omega^2}{\kappa_2^{mn}}.$$

In the case $N_{12} = 0$ substituting the relationships (19) and (29) into the equation of motion (13), the solution is obtained as following

$$\tilde{w}_{mn} = -\frac{R_{mn}}{L_{mn}} A_*, \quad (31)$$

in which

$$R_{mn} = i\rho_1\omega f_{mn} \left(1 + \frac{i\omega}{\kappa_1^{mn} c_1} \right), \quad (32)$$

$$L_{mn} = d_{mn} - \varphi_{mn},$$

$$d_{mn} = d_1^* \lambda_m^4 + (\tilde{d}_1^* + \tilde{d}_2^* + 4d_{12}^*) \lambda_m^2 \lambda_n^2 + d_2^* \lambda_n^4 - N_{11} \lambda_m^2 - N_{22} \lambda_n^2 - q\omega^2. \quad (33)$$

The values of parameters $d_1^*, d_2^*, \tilde{d}_1^*, \tilde{d}_2^*, d_{12}^*$ can be calculated using the formulae in (8), (10) and (12), where $B_s^{(k)}, B_{12}^{(k)}$ are replaced with

$$B_s^{(k)*} = \frac{E_s^{(k)} t_k}{1 - \nu_{12}^{(k)} \nu_{21}^{(k)}} \left(1 + i \frac{\delta_s^{(k)}}{\pi} \right), \quad (34)$$

$$B_{12}^{(k)*} = G_{12}^{(k)} t_k \left(1 + i \frac{\delta_{12}^{(k)}}{\pi} \right).$$

In correspondence with the solution found in (20) amplitudes of sound pressure in points at bounding planes $z = -t_1/2$ and $z = t_1/2 + t_2$ will be equal to

$$\tilde{P}_1 = -i\rho_1\omega \sum_{m,n=1,3,..}^{M,N} \left[f_{mn} \left(1 + \frac{i\omega}{c_1 \kappa_1^{mn}} \right) - i \frac{\omega R_{mn}}{\kappa_1^{mn} L_{mn}} \right] A_*, \quad (35)$$

$$\tilde{P}_2 = \rho_2\omega \sum_{m,n=1,3,..}^{M,N} \frac{R_{mn}}{\kappa_2^{mn} L_{mn}} A_*,$$

For a given value of A_* and the calculated values of \tilde{P}_1, \tilde{P}_2 sound isolation properties of plate and level of sound pressure in half spaces V_1 and V_2 will be described with parameters (p_0 – sound pressure corresponding to the threshold of hearing)

$$R_p^0 = -20 \lg \left| \frac{\tilde{P}_2}{\tilde{P}_1} \right|, \quad R_p^- = -20 \lg \left| \frac{\tilde{P}_1}{p_0} \right|, \quad (36)$$

$$R_p^+ = -20 \lg \left| \frac{\tilde{P}_2}{p_0} \right|,$$

and amplitude values of the dynamic stress components formed at points of the boundary plane of the plate's first layer from by the action of the incident sound wave, will be determined by the formulas

$$\sigma_{11}^{(1)} \Big|_{z=-t_1/2} = -\frac{B_1^{(1)*}}{t_1} \sum_{m,n=1,3,..}^{M,N} \left\{ \frac{H}{B_d} \left[(\tilde{G}_1 - \nu_{21}^{(2)} G_2) \lambda_n^2 - \right. \right. \quad (37)$$

$$\left. \left. - (G_1 - \nu_{21}^{(1)} \tilde{G}_2) \lambda_m^2 \right] - \frac{t_1}{2} (\lambda_m^2 + \nu_{21}^{(1)} \lambda_n^2) \right\} \frac{R_{mn}}{L_{mn}} A_* \dots,$$

which is obtained from (12) using the relationships (9) and solutions (19), (31).

3. Numerical results and their analysis

Based on the solutions obtained for plates made from glass reinforced plastics having the following parameters

$$\begin{aligned} a &= 0.48 \text{ m}, \quad b = 0.56 \text{ m}, \quad t_1 = 2 \text{ mm}, \\ E_1^{(1)} &= 18 \text{ GPa}, \quad E_2^{(1)} = 14 \text{ GPa}, \quad \nu_{12}^{(1)} = 0,3, \\ G_{12}^{(1)} &= 2,9 \text{ GPa}, \\ \delta_1^{(1)} &= 0,03, \quad \delta_2^{(1)} = 0,035, \\ \delta_{12}^{(1)} &= 0,03, \quad r_1 = 3000 \text{ kg/m}^3 \end{aligned}$$

computations have been made in various frequencies f of sound waves to determine the sound isolation parameters R_p^0 , R_p^- , R_p^+ , amplitude values of deflection w_0 and dynamic components of internal bending moment M_{11}^d which is calculated with the formula

$$M_{11}^d = \sum_{m,n=1,3,\dots}^{M,N} \left(d_1 \lambda_m^2 + \tilde{d}_1 \lambda_n^2 \right) \frac{R_{mn}}{L_{mn}} A_* \quad (38)$$

Results of calculations performed for the plate with a damping layer of rubber having the characteristics

$$\begin{aligned} E_1^{(2)} &= E_2^{(2)} = 500 \text{ MPa}, \quad \nu_{12}^{(2)} = \nu_{21}^{(2)} = 0,4, \\ \delta_1^{(2)} &= \delta_2^{(2)} = 1,2, \quad \delta_{12}^{(2)} = 0,9, \quad r_2 = 1500 \text{ kg/m}^3 \end{aligned}$$

in pictures 1,3,5,7 and 9 are introduced with thickness $t_2 = 0; 0,5; 1 \text{ mm}$, while in pictures 2, 4, 6, 8 and 10 with thicknesses $t_2 = 2; 3; 5 \text{ mm}$ (graphs, introduced in pictures 3-10 correspond to $A_* = 1$). Analyzing the obtained results, it can be seen that the damping layer in the plate, having even considerable thickness and logarithmic decrement does not show significant effect on sound isolation properties R_p^0 (in the range $5 \div 10 \text{ dB}$). But, on the other hand, by introducing a damping layer, vibration amplitudes (Fig. 7, 8) and the value of bending moment are (and hence the stress) (Fig. 9, 10) significantly reduced.

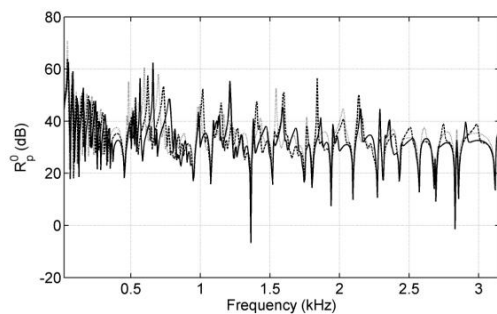


Fig. 1. Dependencies for $R_p^0(f)$:
 ——— $t_2 = 0 \text{ mm}$, ——— $t_2 = 0,5 \text{ mm}$,
 $t_2 = 1 \text{ mm}$

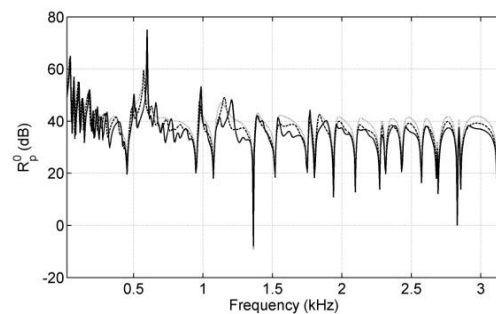


Fig. 2. Dependencies for $R_p^0(f)$:
 ——— $t_2 = 2 \text{ mm}$, ——— $t_2 = 3 \text{ mm}$,
 $t_2 = 5 \text{ mm}$

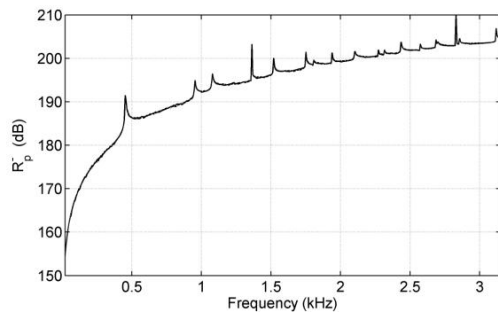


Fig. 3. Dependencies for $R_p^-(f)$:
 — $t_2 = 0$ mm, — $t_2 = 0,5$ mm,
 $t_2 = 1$ mm

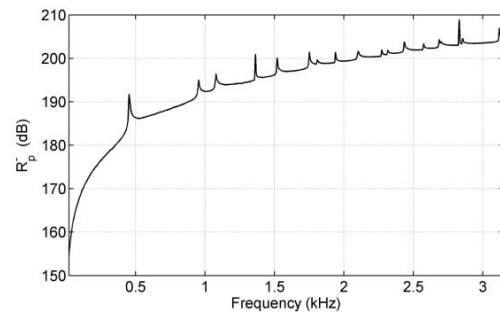


Fig. 4. Dependencies for $R_p^-(f)$:
 — $t_2 = 2$ mm, — $t_2 = 3$ mm,
 $t_2 = 5$ mm

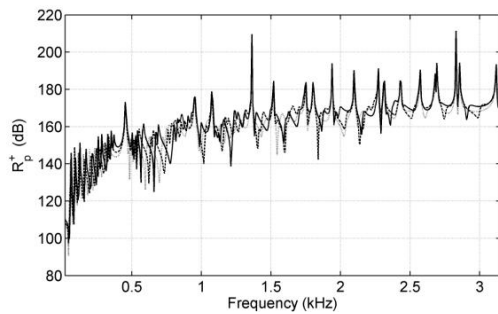


Fig. 5. Dependencies for $R_p^+(f)$:
 — $t_2 = 0$ mm, — $t_2 = 0,5$ mm,
 $t_2 = 1$ mm

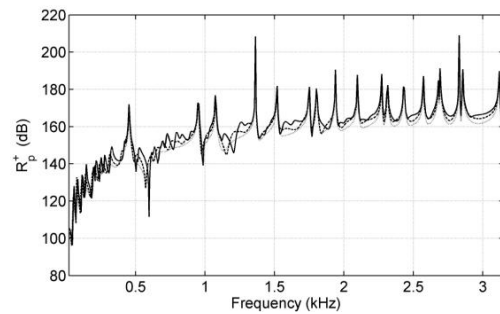


Fig. 6. Dependencies for $R_p^+(f)$:
 — $t_2 = 2$ mm, — $t_2 = 3$ mm,
 $t_2 = 5$ mm

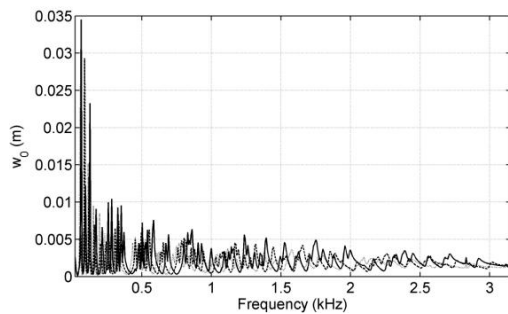


Fig. 7. Dependency of $w_0(f)$:
 — $t_2 = 0$ mm, — $t_2 = 0,5$ mm,
 $t_2 = 1$ mm

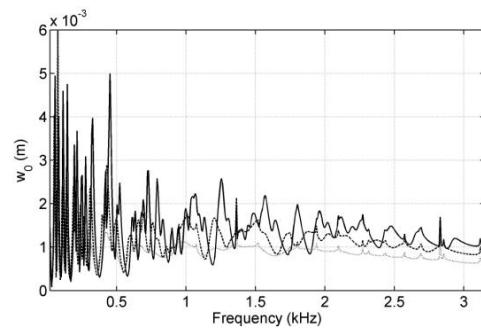
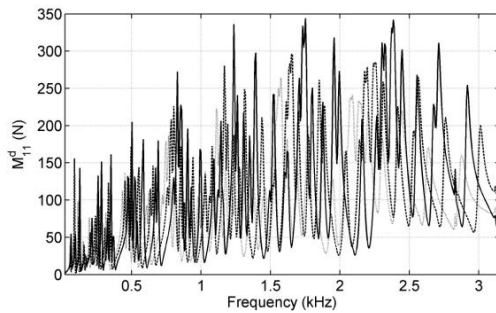
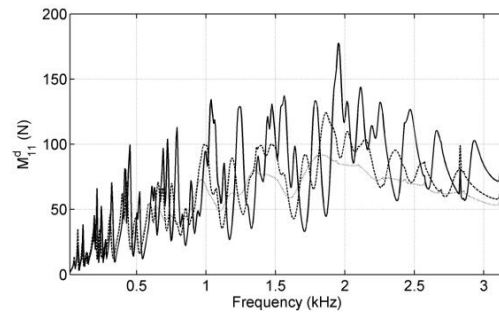


Fig. 8. Dependency of $w_0(f)$:
 — $t_2 = 2$ mm, — $t_2 = 3$ mm,
 $t_2 = 5$ mm

Fig. 9. Dependency of $M_{11}^d(f)$:

—— $t_2 = 0$ mm, ----- $t_2 = 0,5$ mm, $t_2 = 1$ mm

Fig. 10. Dependency of $M_{11}^d(f)$:

—— $t_2 = 2$ mm, ----- $t_2 = 3$ mm, $t_2 = 5$ mm

4. Conclusions

Based on the analysis of the results, we come into a conclusion that the functional rubber-like materials adhesively stuck on thin-walled structural elements as an additional thin layer possessing high internal damping characteristics can significantly reduce the amplitude values of deformations and deflections of structural elements in the resonant modes of loading thereby forming a reduced level of radiated sound pressure. The use of these special coatings with high logarithmic decrement ratios leads to dramatic reduction in the level of cyclic stresses resulting in structural elements, as a consequence, to a dramatic increase in durability of structure.

Acknowledgements

The study was performed by the financial support of Russian Scientific Foundation under project no. №14-19-00667.

References

- [1] Munin A. G., Samokhin V.F. (1986) *Aviatsionnaya akustika. Chast'1: Shum na mestnosti dozvukovykh passazhirskikh samoletov i vertoletov* [Aviation acoustics. Part 1: Noise in the area of subsonic passenger aircraft and helicopters]. Moscow, Mashinostroenie Pub., 243p. (in Russian).
- [2] Munin A. G., Efimtsov B.M., eds. (1986) *Aviatsionnaya akustika. Chast'2: Shum v salonakh passazhirskikh samoletov* [Aviation acoustics. Part 2: noise in the cabin of passenger aircraft]. Moscow, Mashinostroenie Pub., p.261 (in Russian)
- [3] Grigolyuk E.I. (1970) *Problemy vzaimodeystviya obolochek s zhidkost'yu* [Problems of interaction between shells and liquid] *Trudy VII Vsesoyuznoy konferentsii po teorii obolochek i plastin* [Proc. of 7th All-Union Conference on the Theory of Shells and Plates]. Moscow, Nauka Pub., pp. 755-778. (in Russian)
- [4] Grigolyuk E.I., Gorshkov A.G. (1974) *Nestatsionarnaya gidrouprugost' obolochek* [Unsteady hydroelasticity of shells]. Leningrad, Sudostroenie Pub., p.208 (in Russian)
- [5] Gorshkov A. G. (1980) *Vzaimodeystvie udarnykh voln s deformiruemyimi pregradami* [Interaction of shock waves with a deformable barrier]. *Itogi nauki i tekhniki VINITI. Mekhanika deformiruemogo tverdogo tela* [Results in science and technology of VINITI. Mechanics of deformable solids], no. 13, pp. 105-186. (in Russian)
- [6] Guz' A.N., Kubenko V.D. (1982) *Metody rascheta obolochek. Tom 5. Teoriya nestatsionarnoy aerogidrouprugosti obolochek* [Methods of analysis of shells. Volume 5. Theory unsteady aerohydroelasticity shells]. Kiev. Nauk. dumka Pub!., 400 p. (in Russian)
- [7] Bersudskiy B.E., Krisin V.N., Lesnikh S.I. (1975), *Technologiya izgotovleniya sotovykh aviatsionnykh konstruksij* [Manufacturing technology of honeycomb aviation constructions], Moscow, Mashinostroenie Pub., p216 (in Russian).
- [8] Endogur A.I., Weinberg M.B., Jarusalemiskij K.M. (1986), *Sotovie konstruksii, vibor parametrov i proektirovanie*, [Honeycomb structures, selection of parameters and design], Moscow, Mashinostroenie Pub., p200 (in Russian).
- [9] Panovko Ya. G. (1960) *Vnutrennee trenie pri kolebaniyax uprugikh sistem* [Internal friction in the vibration of elastic systems], Moscow, Gos. Pub. Phys.Mat. Lit. p193 (in Russian).